The flow regime studied above can also be singled out in the more general case when instead of the linear velocity profile at the bottom of the boundary layer (1.4), a power-law profile $\Psi_1 Y(x_1, Y \to 0) = \lambda Y^k$, where $0 < k < \infty$, holds. The flow will be a creeping flow if

$$2 > n > \begin{cases} 3/(k+1) & \text{for } 1/2 < k < 2, \\ 1 & \text{for } k > 2. \end{cases}$$

The author is very grateful to A. V. Zubtsov and V. F. Molchanov for fruitful discussions.

LITERATURE CITED

1. N. E. Zhukovskii and S. A. Chaplygin, "Rubbing of a lubricating layer between a pin and a bearing," Tr. Otdeleniya Fizicheskikh Nauk Obshchestva Lyubitelei Estestvoznaniya, 13, No. 1 (1906).
2. P. M. Morse and H. Feshbach, Methods of Theoretical Physics [Russian translation], Inostr. Lit., Moscow (1960), Vol. II.
3. F. D. Gakhov, Boundary-Value Problems, Nauka, Moscow (1977).
4. A. V. Zubstov and V. I. Ponomarev, "Asymptotic solution of the problem of a two-dimensional flow of viscous fluid over a fluted surface," Uch. Zap. TsAGI, 3, No. 2 (1972).
5. V. V. Sychev, "Laminar detachment," Izv. Akad. Nauk SSSR, MZhG, No. 3 (1972).
6. S. K. Betyaev, "Detached flows," Preprint No. 14-83, Institute of Theoretical and Applied Mathematics, Siberian Branch of the USSR Academy of Sciences, Novosibirsk (1983).

PENETRATION OF A BLUNT BODY INTO A SLIGHTLY COMPRESSIBLE LIQUID

A. A. Korobkin

UDC 532.59:522.2

Introduction. We will consider the initial stage of nonsteady state motion of a liquid produced by its penetration by a solid body. Initially (t' = 0) the liquid is at rest and the body touches the free surface at a single point. The region $\Omega(t')$, occupied by the liquid, varies with time, while its boundary $\partial\Omega(t')$ consists of the free surface Σ_1, and the solid surface of the penetrating body Σ_2, the contact line between them Γ, and, possibly, the inmobile solid walls Σ_3 (as for example, in landing of an airplane on the surface of a body of water). The velocity range is assumed such that the Reynolds number Re >> 1 while the Mach number M << 1.

Quantitative information on the penetration process can be obtained only from numerical calculations. However, the accuracy of such calculations decreases at times when the flow topology changes, singularities develop in the pressure field, infinite accelerations of liquid particles occur, etc. Singularities like these must be treated analytically. Numerical solution of the problem of penetration of sharp bodies (wedges, cones) into an incompressible liquid were constructed in [1], and the pressure distribution obtained for the contact spot agreed well with experiment. But for blunt bodies use of the ideal incompressible liquid model leads to infinite pressures at t' = 0, no matter how low the penetration velocity [2]. This is because the incompressible liquid model in which the perturbation propagation velocity is assumed infinite is not capable of describing the important stage of the process of penetration of a blunt body. In fact there exists a time t'_* of the order of several μsecs, such that at $t' < t'_*$ the contact line Γ moves with a velocity exceeding the speed of sound in the liquid. The perturbation front is then attached to the line Γ, and the perturbed portion of the liquid is limited by the solid surface on one side and the shock wave

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 104-110, September-October, 1984. Original article submitted September 29, 1983.

0021-8944/84/2505-0751$08.50

front on the other. Until the shock wave breaks away from the contact line the free surface remains undisturbed. Thus, to obtain realistic results in this stage of the calculations it is necessary to use a compressible liquid model, independent of the value of the Mach number M. The solutions constructed and studied in [3] for small times t' for penetration of a blunt body into an ideal incompressible liquid describe the process well for $t' \gg t'_*$, when the shock wave has departed sufficiently from the contact line. As will be shown below, this solution should be considered as the major term in an external (relative to t' = 0) asymptotic expansion as $M \to 0$ of the solution of the problem of penetration into an ideal compressible liquid.

1. Problem Formulation. We will limit our study to the planar case of penetration of a parabolic contour.

We will consider the planar nonsteady-state isentropic motion of an ideal compressible liquid, which at the moment t' = 0 fills the semiplane y' < 0 and is initially at rest (as before, the primes denote dimensional variables). The line y' = 0 is the free surface at the initial moment. It is assumed that surface tension and external mass forces are absent.

Let R and V be positive constants. For a fixed t' the equation

$$y' = (1/2R)x'^2 - Vt' \tag{1.1}$$

defines a parabola in the plane x', y', which we will identify with the rigid solid contour.

At t' = 0 this contour is tangent to the free surface at x' = 0. Equation (1.1) specifies the motion of the contour along the y' axis at a constant velocity V. We must specify the liquid motion which then develops, assuming that the portion of its boundary which is not a portion of the solid contour remains free. In the plane of the Lagrangian coordinates ξ', η' the region occupied by the liquid is known beforehand — it is the semiplane $\eta' < 0$.

We choose as the length scale the radius R of the parabola of Eq. (1.1) at the point x' = 0, while for a time scale factor we use the quantity R/V, and transform to dimensionless variables (which are denoted by the absence of primes).

Since motion commences from a state of rest and external mass forces are absent, by Lagrange's theorem [4] the flow of the ideal compressible liquid will be nonturbulent. Consequently, there exists a potential for the velocity $\varphi_0'(x', y', t')$, such that $x_t = \varphi_{0x}$, $y_t = \varphi_{0y}$, where $\varphi_0' = RV\varphi_0(x, y, t)$. The function φ_0 satisfies the equation [5]

$$\Delta\varphi_0 = S(\rho)\left(\varphi_{0tt} + 2\nabla\varphi_0\nabla\varphi_{0t} + \nabla\varphi_0\nabla\left(\frac{1}{2}|\nabla\varphi_0|^2\right)\right),\ S(\rho) = V^2c^{-2}(\rho) \tag{1.2}$$

($\rho(\xi, \eta, t)$ is the liquid density, $c(\rho)$ is the local speed of sound). We combine with Eq. (1.2) the boundary conditions (on the free surface Σ_1 the pressure p is constant, on the contact spot Σ_2 the nonpenetration condition is satisfied), and initial conditions ($\varphi_0 = 0$, $\varphi_{0t} = 0$ at t = 0).

We introduce the Lagrangian coordinates ξ, η in which the flow region is fixed in a manner such that $x = \xi$, $y = \eta$ at t = 0. In these variables Eq. (1.2) appears as

$$\begin{gathered} S(\rho)\varphi_{tt} - \Delta\varphi_0 = S(\rho)L(\varphi) \quad \text{for} \quad \eta < 0, \\ \Delta\varphi_0 = (N^{*-1}\nabla_\xi)(N^{*-1}\nabla_\xi\varphi),\ \mathbf{x}_t = N^{*-1}\nabla_\xi\varphi, \end{gathered} \tag{1.3}$$

where $\varphi(\xi, \eta, t) = \varphi_0(\mathbf{x}(\xi, \eta, t), t)$; $N = \partial(\mathbf{x})/\partial(\boldsymbol{\xi})$ is the Jacobi matrix; L is a nonlinear differential operator; $\mathbf{x} = (x, y)$; $\boldsymbol{\xi} = (\xi, \eta)$. We denote by $a(t)$ the mapping into Lagrangian coordinates of the contact points Σ_1 and Σ_2. Then at any t in some interval [0, T] the line $\eta = 0$ bounding the liquid consists of three regions: $\xi < -a(t)$, $|\xi| \leqslant a(t)$, $\xi > a(t)$, where the function $a(t)$ must be determined. The sections $|\xi| > a(t)$ are free boundaries.

We will define the boundary conditions which must be satisfied by the unknown function $\varphi(\xi, t)$. The Bernoulli integral for the potential flow of a compressible liquid has the form

$$\varphi_{0t} + (1/2)q^2 + i = 0, \tag{1.4}$$

where $q^2 = |\nabla \varphi_0|^2$; i is the enthalpy related to the pressure p and the density ρ by the expression $di = \rho^{-1}dp$. Considering that $\varphi_t = \varphi_{0t} + q^2$, we rewrite Eq. (1.4) in Lagrangian coordinates:

$$\varphi_t = (1/2)q^2 - i.$$

Since on the free surface p = const, and consequently i = const also, we have

$$\varphi_t = (1/2)q^2 \quad \text{for} \quad \eta = 0, \quad |\xi| > a(t) \tag{1.5}$$

(we recall that the potential φ is defined to the accuracy of a constant term).

On the contact spot Σ_2 we substitute the nonpenetration condition

$$(N^{*-1}\nabla_\xi\varphi - \mathbf{v})\,\mathbf{n} = 0, \quad |\xi| < a(t), \ \eta = 0, \tag{1.6}$$

where $\mathbf{v} = (0, -1)$; $\mathbf{n}$ is the normal to the surface Σ_2. We add to the problem the condition at infinity

$$\varphi \to 0 \quad \text{as} \quad |\xi| \to \infty, \tag{1.7}$$

initial conditions

$$\varphi = 0, \ \varphi_t = 0 \quad \text{at} \quad t = 0 \tag{1.8}$$

and require that liquid particles lying on the free surface Σ_1 not penetrate through the surface of the solid contour over the entire time of the motion [3]

$$Y \leqslant \frac{1}{2}(\xi + X)^2 - t \quad \text{for} \quad \eta = 0, \ |\xi| > a(t), \tag{1.9}$$

where $\mathbf{X} = \mathbf{x} - \xi$, $\mathbf{X} = (X, Y)$.

The problem thus formulated is complicated by nonlinearity and the presence of an unknown boundary for change in form of the boundary condition (we recall that $a(t)$ must be determined in the course of solution of Eqs. (1.3), (1.5)-(1.8) with the additional single-sided limitation Eq. (1.9) on displacement of free surface particles).

2. Asymptotic Solution. We will seek a solution of Eqs. (1.3), (1.5)-(1.9) in the form of an expansion in powers of the parameter M, where $M = V/c_0$, c_0 being the speed of sound in the liquid at rest, as $M \to 0$:

$$\varphi(\xi, t) = \varphi^{(0)}(\xi, t) + M^2\varphi^{(1)}(\xi, t) + \ldots, \tag{2.1}$$
$$S(\rho) = M^2 + \varepsilon_1(M)s_1(\rho) + \ldots, \quad a(t) = a^{(0)}(t) + \delta_1(M)a^{(1)}(t) + \ldots,$$

$\{\varepsilon_i(M)\}$, $\{\delta_i(M)\}$ are asymptotic sequences while $\varepsilon_i(M) = o(M^2)$ for $i \geqslant 1$. Then the problem for the main term of the asymptote of the velocity potential as $M \to 0$ has the form

$$\begin{gathered}
(N_0^{*-1}\nabla_\xi)(N_0^{*-1}\nabla_\xi\varphi^{(0)}) = 0 \quad \text{for} \quad \eta < 0, \\
\varphi_t^{(0)} = \frac{1}{2}|N_0^{*-1}\nabla_\xi\varphi^{(0)}|^2 \quad \text{for} \quad \eta = 0, \ |\xi| > a(t), \\
(N_0^{*-1}\nabla_\xi\varphi^{(0)} - \mathbf{v})\cdot\mathbf{n} = 0 \quad \text{for} \quad \eta = 0, \ |\xi| < a(t), \\
\varphi^{(0)}, \varphi_t^{(0)} = 0 \quad \text{for} \quad t = 0, \\
\varphi^{(0)} \to 0 \quad \text{for} \quad |\xi| \to \infty, \\
Y^{(0)} \leqslant \frac{1}{2}(\xi + X^{(0)})^2 - t \quad \text{for} \quad \eta = 0, \ |\xi| > a(t)
\end{gathered} \tag{2.2}$$

and, moreover,

$$N_0 = I + \partial(X^{(0)}, Y^{(0)})/\partial(\xi, \eta), \quad \mathbf{X}_t^{(0)} = N_0^{*-1}\nabla_\xi\varphi^{(0)},$$

where I is a unit matrix. Problem (2.2), which describes penetration of a solid contour into an ideal liquid, was studied in [3] for the initial stage of penetration ($t \to 0$). As

$t \to 0$ the asymptotes of the unknown functions are given by

$$\varphi^{(0)}(\xi, \eta, t) = \mathrm{Im}\,(\sqrt{\zeta^2 - a^2(t)} - \zeta)(1 + O(\sqrt{t})),\ \zeta = \xi + i\eta,$$
$$a^{(0)}(t) = 2\sqrt{t}(1 + O\sqrt{t})), \tag{2.3}$$
$$p^{(0)}(\xi, 0, t) = \frac{2}{\sqrt{a^2(t) - \xi^2}}(1 + O(\sqrt{t})) \quad \text{for} \quad |\xi| < a(t).$$

Zeroth approximation (2.3) satisfies Eq. (2.2) to the accuracy of $O(\sqrt{t})$ everywhere except narrow zones near the contact points, the size of which is the order of magnitude of $t^{3/2}$ as $t \to 0$. Within these zones the flow pattern was refined in [3].

It has already been noted that the main term of the pressure asymptote $p^{(0)}$ has a singularity at the point $t = 0$ as $M \to 0$, i.e., expansion (2.1), considered as asymptotic, loses force in the immediate vicinity of the point $t = 0$, which, as will be shown below, is of order $O(M^2)$ as $M \to 0$. To refine the flow structure within this vicinity it is necessary to construct internal expansions. It is in just this region that the flow characteristics are dominant for the problem as a whole.

3. Internal Expansion. We will define the internal variables α, β, τ with the expressions $\xi = \delta_1(M)\alpha$, $\eta = \delta_1(M)\beta$, $t = \delta_0(M)\tau$ and seek an internal expansion of the solution of Eqs. (1.3), (1.5)-(1.8) in the form

$$\varphi(\xi, \eta, t) = \varepsilon_0(M)\Psi^{(0)}(\alpha, \beta, \tau) + \varepsilon_1(M)\Psi^{(1)}(\alpha, \beta, \tau) + \ldots, \tag{3.1}$$
$$a(t) = \delta_1(M)b(\tau) + \delta_2(M)b^{(2)}(\tau) + \ldots,$$

where $\{\varepsilon_i(M)\}_{i=0}^{\infty}$, $\{\delta_i(M)\}_{i=1}^{\infty}$ are asymptotic sequences as $M \to 0$. According to the principle of merging asymptotic solutions [6], it follows from Eq. (2.3), first, that

$$\tau^{-1/2}b(\tau) \to 2\sqrt{\delta_0(M)}/\delta_1(M) \quad \text{as} \quad \tau \to \infty,$$

whence we define $\delta_1(M) = \sqrt{\delta_0(M)}$, and second, that

$$\Psi^{(0)}/\mathrm{Im}(\sqrt{z^2 - b^2(\tau)} - z)^2 \to \delta_1(M)/\varepsilon_0(M) \quad \text{as} \quad \tau \to \infty, \tag{3.2}$$

where $z = \alpha + i\beta$. From condition (3.2) we obtain

$$\varepsilon_0(M) = \delta_1(M) = \sqrt{\delta_0(M)}.$$

Substituting Eq. (3.1) in Eq. (1.3) and conditions (1.5)-(1.9), we retain the main terms as $M \to 0$. The condition of nontriviality of the solution of the problem thus obtained leads to the requirement $\delta_1(M) = M$. The main term of the asymptote of the external expansion of the velocity potential Φ as $M \to 0$ satisfies the relationships

$$\Psi^{(0)}_{\tau\tau} - \Delta\Psi^{(0)} = 0 \quad \text{for} \quad \beta < 0,$$
$$\Psi^{(0)}_{\beta} = -1 \quad \text{for} \quad \beta = 0,\ |\alpha| \leqslant b(\tau), \tag{3.3}$$
$$\Psi^{(0)}_{\tau} = 0 \quad \text{for} \quad \beta = 0,\ |\alpha| > b(\tau),$$
$$\Psi^{(0)} = 0,\ \Psi^{(0)}_{\tau} = 0 \quad \text{for} \quad \tau = 0.$$

From the merger principle it follows that

$$b(\tau) \backsim 2\sqrt{\tau} \quad \text{as} \quad \tau \to \infty, \tag{3.4}$$

so that Eq. (3.2) is satisfied automatically (it will be sufficient to consider the asymptote of the solution of Eqs. (3.3), (3.4) at large τ). We note that $b(\tau) = \sqrt{2\tau}$ for $\tau \leqslant \tau_*$, where τ_* is the moment of shock wave exit onto the free liquid surface.

From inequality (1.9) and nonpenetration condition (1.6) for $\tau > \tau_*$ it follows that

$$Y = \frac{1}{2}(\xi + X)^2 - t \quad \text{for} \quad \eta = 0,\ \xi = a(t) + 0,\ \tau_* M^2 < t < T_1, \tag{3.5}$$

where T_1 is the moment of breakoff of the rotation region of the liquid free surface from the body surface. We recall that X, Y are the displacements of liquid particles along the ξ, η axes, respectively. In the internal variables α, β, τ for the zeroth approximation of the velocity potential $\Psi^{(0)}(\alpha, \beta, \tau)$ we obtain from Eq. (3.5)

$$\int\limits_{b(\tau)-1/2}^{\tau} \Psi_{\beta}^{(0)}(b(\tau), 0, s)\, ds = \frac{1}{2} b^2(\tau) - \tau, \tag{3.6}$$

where the left side of the equation is the displacement of a liquid particle along the β axis over the time τ (in the zeroth approximation displacement of free surface particles along the α axis is absent). Simultaneous solution of Eq. (3.3) and supplementary equation (3.6) gives the main term of the asymptote of the velocity potential as $M \to 0$. Equation (3.4) is then satisfied automatically (it is sufficient to consider the asymptote of the solution of Eqs. (3.3), (3.6) for large τ).

We will seek the quantity τ_* in the form

$$\tau_* = \tau_*^{(0)} + \chi_1(M)\tau_*^{(1)} + \ldots,$$

where $\{\chi_i(M)\}_{i=1}^{\infty}$ is an asymptotic sequence as $M \to 0$. The moment of exit of the shock wave onto the free surface is characterized by the fact that at $\tau = \tau_*$ the velocity of shock wave motion W' in the vicinity of the contact point and the velocity of motion of the contact point itself along the free surface coincide: $b_\tau(\tau) = W'/c_0$ at $\tau = \tau_*$. The zeroth approximation of this condition ($W' \to c_0$ as $M \to 0$) gives $\tau_*^{(0)} = 1/2$.

We will study the pressure distribution on the contact spot at $\tau \leqslant 1/2$ (the function $b(\tau)$ is known beforehand). We introduce the following notation:

$$u(\alpha, \tau) = \Psi^{(0)}(\alpha, 0, \tau), \quad w(\alpha, \tau) = \Psi_\beta^{(0)}(\alpha, 0, \tau),$$
$$D = \{(\alpha, \tau) \mid |\alpha| < b(\tau)\},$$
$$p' = \rho_0 c_0 V(q^{(0)}(\alpha, \beta, \tau) + Mq^{(1)}(\alpha, \beta, \tau) + \ldots),$$

where p' is the pressure and c_0 is the velocity of sound in the liquid at rest. The function $u(\alpha, \tau)$ is related to the zeroth approximation for the pressure $q^{(0)}(\alpha, 0, \tau)$ by the expression

$$q^{(0)} = -u_\tau(\alpha, \tau), \tag{3.7}$$

and to $w(\alpha, \tau)$ by the expression [7]

$$u(\alpha, \tau) = \frac{1}{\pi} \iint\limits_{\sigma(\alpha,\tau)} \frac{w(x, t)\, dx dt}{\sqrt{(\tau - t)^2 - (x - \alpha)^2}}, \tag{3.8}$$

where $\sigma(\alpha, \tau) = \{x, \ t \mid \alpha - \tau < x < \alpha, \ 0 \leqslant t \leqslant x - \alpha + \tau\} \cup \{x, \ t \mid, \ \alpha < x < \alpha + \tau, 0 \leqslant t \leqslant \alpha + \tau - x\}$. If $(\alpha, \tau) \in D \cap \sigma(0, 3/2)$, then in Eq. (3.8) the integration is performed over the region $D \cap \sigma(\alpha, \tau)$ in which $w(\alpha, \tau) = -1$. With the aid of Eq. (3.8) we construct $u(\alpha, \tau)$, and thus, the main term of the pressure asymptote $q^{(0)}$ at $\beta = 0$ $|\alpha| < \sqrt{2\tau}, \tau \leqslant 1/2$. It follows from Eqs. (3.7), (3.8) that

$$q^{(0)}(\alpha, 0, \tau) = \frac{1}{\pi\zeta(\alpha, \tau)} K\left(\frac{1}{\sqrt{2}} \sqrt{1 - \frac{1/2 - \tau}{\zeta(\alpha, \tau)}}\right), \tag{3.9}$$
$$\zeta(\alpha, \tau) = \sqrt{(\tau + 1/2)^2 - \alpha^2}, \ |\alpha| \leqslant \sqrt{2\tau}, \ \tau \leqslant 1/2,$$

where $K(x)$ is a full elliptical integral of the first sort. We will present some special cases of Eq. (3.9):

1) $q^{(0)}(0, 0, 0) = 1$, i.e., at the moment $t = 0$ the pressure is equal to the hydraulic shock pressure $p' = \rho_0 c_0 V$;

2) $q^{(0)}(\alpha, 0, 1/2) = K(2^{-1/2})\pi^{-1}(1 - \alpha^2)^{-1/2}$ for $\tau = \tau_*^{(0)}$;

3) $q^{(0)}(\alpha, 0, \tau) = (1 - 2\tau)^{-1}$ for $\alpha = \sqrt{2\tau}, \ \tau \leqslant \tau_*^{(0)}$.

Equation (3.9) indicates that with increase in τ the pressure distribution in the contact region becomes ever more nonuniform, and up to the time $\tau_*^{(0)} = 1/2$ the expansions of Eq. (3.1) lose force in the vicinity of the point $\tau = 1/2$, $\alpha = 1$, $\beta = 0$.

We will consider the pattern of perturbation wave motion. The shock front is the envelope of the family of curves

$$\beta^2 + (\alpha - \lambda)^2 = (\tau - \lambda^2/2)^2, \ \beta < 0, \ |\lambda| < \sqrt{2\tau}, \ \tau \leqslant \tau_*,$$

which can be specified parametrically in the form [8]

$$\alpha = \lambda(\tau + 1 - \lambda^2/2), \ \beta = -(\tau - \lambda^2/2)\sqrt{1 - \lambda^2}, \tag{3.10}$$

$$|\lambda| < \sqrt{2\tau}, \ \tau \leqslant \tau_*,$$

where λ is a parameter. Then the equation of the tangent to the shock wave front at the contact point $\alpha = \sqrt{2\tau}$, $\beta = 0$ has the form

$$\beta = \sqrt{\frac{2\tau}{1-2\tau}}(\alpha - \sqrt{2\tau}).$$

Consequently, with increase in τ, the angle formed by the tangent to the shock front at the point $(\sqrt{2\tau}, 0)$ with the α axis increases from zero at $\tau = 0$ to $\pi/2$ at $\tau = 1/2$.

On the shock front the conditions of mass and momentum conservation are given by [9]

$$\rho'(W' - u') = \rho_0(W' - u_0), \ p' - p_0 = \rho_0(W' - u_0)(u' - u_0),$$

where W' is the speed of shock wave propagation in water with initial pressure p_0, density ρ_0, and flow velocity u_0 (the prime denoting dimensional quantities). In our case $p_0 = 0$, $u_0 = 0$, so that

$$W' = \sqrt{\frac{p'}{\rho' - \rho_0}\frac{\rho'}{\rho_0}}, \ u' = W'(1 - \rho_0/\rho'). \tag{3.11}$$

The equation of state for water at sufficiently low pressure may be written in the form [9]

$$p' = B[(\rho'/\rho_0)^n - 1],$$

where $B \simeq 3.085 \cdot 10^8$ Pa, $n \simeq 7.15$. Consequently, the speed of sound c' is related to the water density behind the shock wave front in the following manner:

$$c'^2 = dp'/d\rho' = c_0^2(\rho'/\rho_0)^{n-1}, \ c_0^2 = Bn/\rho_0$$

or $c' = c_0(1 + nM^2p)^{(n-1)/2n}$. Here $p' = \rho_0 V^2 p$, and V is the velocity of the penetrating body.

It is natural to assume that $M^2p \to 0$ as $M \to 0$ over the entire flow region. Then $c' \to c_0$, $W' \to c_0$, and Eq. (3.9) gives the asymptote of the pressure distribution on the contact spot as $M \to 0$ $(\tau \leqslant \tau_*)$. We note that the shock wave must exit onto the free liquid surface at $\tau_* < 1/2$, since pressure increase behind the shock wave front (see Eq. (3.9)) leads to an increase in the velocity of its motion W' according to Eq. (3.11).

We will seek the value of the pressure p_* at the contact point at $\tau = \tau_*$ in the form

$$p_* = \nu_0(M)\Pi_0 + \nu_1(M)\Pi_1 + \ldots, \tag{3.12}$$

where $\{\nu_i(M)\}_{i=0}^{\infty}$ is an asymptotic sequence as $M \to 0$. Substituting expansion (3.12) in the first expression of Eq. (3.11), we obtain

$$W' = c_0\left(1 + \frac{n+1}{4}M^2\nu_0(M)\Pi_0 + \ldots\right) \quad \text{for} \quad \tau = \tau_*.$$

Then the condition for shock wave exit onto the free surface $(2\tau_*)^{-1/2} = W'(\tau_*)/c_0$ in the first approximation leads to the relationship

$$-\chi_1(M)\tau_*^{(1)} = \frac{n+1}{4}M^2\nu_0(M)\Pi_0. \tag{3.13}$$

As $\tau \to \tau_*$, on the shock wave front, the position of which is given by Eq. (3.10), Eq. (3.9) gives

$$M\nu_0(M)\Pi_0 = -\left(2\chi_1(M)\tau_*^{(1)}\right)^{-1}. \tag{3.14}$$

Nontriviality of Eqs. (3.13), (3.14) requires that $\chi_1 = M^2\nu_0$, $M\chi_1\nu_0 = 1$ or $\nu_0(M) = M^{-3/2}$, $\chi_1(M) = \sqrt{M}$. Solving Eqs. (3.13), (3.14) with consideration of the expressions obtained for the functions $\nu_0(M)$, $\chi_1(M)$, we will have $\tau_*^{(1)} = -\sqrt{(n+1)/8}$, $\Pi_0 = \sqrt{2/(n+1)}$. Consequently, for small M the maximum pressure occurs at $t_* = (M^2/2)(1 + O(\sqrt{M}))$, $\xi_* = M(1 + O(\sqrt{M}))$ and its value is given by

$$p'_* = \sqrt{2/(n+1)}\,\rho_0 c_0^{3/2} V^{1/2}(1 + O(\sqrt{M})).$$

Thus, as $M \to 0$ the pressure at the contact points at time τ_* is much greater than the initial shock pressure. Of course the dimensional pressures remain finite and tend to zero together with the penetration velocity V.

From the expressions presented above it is simple to obtain an expression for the liquid velocity behind the shock wave front u' at time τ_*:

$$u'_* = \sqrt{2/(n+1)}\,c_0^{1/2} V^{1/2}(1 + O(\sqrt{M})).$$

Consequently, even at low penetration velocities V the velocity u'_* may be quite high.

Equation (3.9) indicates that at $\tau < \tau_*$ the kinetic energy of the solid body is partially transformed into elastic energy of the compressed liquid and accumulated therein. At time τ_* the free surface of the liquid deforms, creating a counterflow. The elastic energy of the compressed liquid is transformed into kinetic energy of the flow. If we consider the function $b(\tau)$ known, then problem (3.3) is equivalent to the problem of supersonic flow around a thin wing with sharp edges [7], where the Mach number is equal to two. Using the method proposed in [7] it can be shown that at $\tau > \tau_*$ with increase in τ the pressure decreases at each point of the contact spot.

LITERATURE CITED

1. M. Shiffman and D. C. Spencer, "The force of impact on a cone striking a water surface (vertical entry)," Commun. Pure Appl. Math., 379, No. 4 (1951).
2. J. P. Moran and K. P. Kerney, "On the small-perturbation theory of water-exit and entry," in: Developments in Mechanics, Vol. 2, Pergamon, Oxford (1963).
3. V. V. Pukhnachov and A. A. Korobkin, "Initial asymptotics in problems of blunt body entrance into liquid," in: Third International Conference on Numerical Slip Hydrodynamics, Paris (1981).
4. N. E. Kochin, N. A. Kibel', and N. V. Roze, Theoretical Hydrodynamics, Part I [in Russian], Fizmatgiz, Moscow (1963).
5. L. V. Ovsyannikov, Lectures on the Fundamentals of Gas Dynamics [in Russian], Nauka, Moscow (1981).
6. J. Cole, Perturbation Methods in Applied Mechanics [Russian translation], Mir, Moscow (1972).
7. E. A. Krasil'shchikova, "Effect of turbulent shrouds on steady state motion of a wing," Dokl. Akad. Nauk SSSR, 58, No. 6 (1947).
8. G. Korn and T. Korn, Mathematical Handbook for Scientists and Engineers, 2nd edn., McGraw-Hill (1967).
9. O. A. Berezin and L. A. Grib, "Irregular reflection of a plane shock wave in water from a free surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1960).